Constraining sterile neutrino dark matter by phase-space density observations

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Abstract

We apply phase-space density considerations to obtain lower bounds on the mass of sterile neutrino as dark matter candidate. The bounds are different for non-resonant production, resonant production in the presence of lepton asymmetry and production in decays of heavier particles. In the former case our bound is comparable to, but independent of the Lyman-α bound, and together with X-ray upper limit it disfavors non-resonantly produced sterile neutrino dark matter. An interesting feature of the latter case is that *warm* dark matter may be composed of *heavy* particles.

One candidate for dark matter particle is sterile neutrino. In the first place, the existence of sterile neutrinos is favored by the discovery of neutrino oscillations. Furthermore, by adding sterile neutrinos to the Standard Model and fine-tuning parameters in the neutrino sector, one is able to explain all known facts in high energy physics and cosmology without introducing any other new fields except for inflaton [1, 2, 3].

The sterile neutrino dark matter should satisfy cosmological and astrophysical constraints. These constraints depend on the way sterile neutrinos are produced in the early Universe. There are several mechanisms capable of generating sterile neutrino in right amount. These include the non-resonant production via active-sterile neutrino mixing [4], resonant production in the presence of lepton asymmetry [5, 6], production in decays of heavier particles (see, e.g., Refs. [3, 7] in the context of νMSM model) and production in scattering (see e.g., Ref. [8]). In this paper we consider astrophysical constraints on sterile neutrino dark matter generated by the non-resonant production mechanism. We comment on other production mechanisms towards the end of this paper.

Sterile neutrino is coupled to active ones through mixing only. Thus, the mixing angle controls the decay rate of sterile neutrino into active neutrino and photons. These decays in galactic halos would produce X-ray emission lines, which are severely constrained [9] by

current observations. Within the non-resonant production mechanism, the mixing angle and the mass m of sterile neutrino are related to each other by the requirement that sterile neutrinos make all of the dark matter. So, the X-ray observations put the upper bound on the mass, $m < 4\,\mathrm{keV}$ [10].

Sterile neutrino with the mass in the keV range is *warm* dark matter candidate. In fact, warm dark matter may be even more attractive than cold dark matter [11], as it might be able to address emerging problems of the standard CDM cosmology on small scales, such as missing satellites [12], galactic density profiles [13] and angular momentum of spiral galaxies [14]. However, like any warm dark matter candidate sterile neutrino should not be too "warm" in order to form observed small scale structures. This implies that its mass should not be too small.

Well known lower limits on sterile neutrino mass come from the observations of Lyman-α forest — multiple absorption lines in spectra of distant quasars [15, 16]. Most of these lines correspond to smooth overdense regions of warm ionized intergalactic medium which are believed to trace dark matter clustering. To derive the sterile neutrino mass bounds, one compares the statistical properties of Lyman-α absorption lines with those predicted within various cosmological models by semi-analytical methods and numerical simulations. Recent Lyman-α lower bounds on the mass of sterile neutrino dark matter produced via the non-resonant production mechanism are $m > 5.6 - 28\,\mathrm{keV}$ depending on data set [16]. Together with the X-ray upper bound they disfavor the non-resonantly produced sterile neutrino as the dark matter candidate.

Given the uncertainties in the Lyman-α constraints, it is worth deriving a lower bound on sterile neutrino mass in an independent way. In this paper we employ the Tremaine–Gunn approach, which was successfully used nearly thirty years ago to rule out the scenario of active neutrino hot dark matter [17]. This approach relies on the fact that coarse-grained distribution function of collisionless particles decreases in the course of evolution.

The process of gravitational formation of compact objects from collisionless dark matter particles is described by the Vlasov equations governing the evolution of distribution function in self-consistent gravitation field, see, e.g. Ref. [18] and references therein. One of the most important phenomena taking place in such systems is mixing of distribution function in phase-space. The phase-space volume occupied by the system is invariant under Vlasov equation dynamics. Nevertheless, particle trajectories chaotically disperse, exploring new areas of phase-space and forming there ever more complicated fine structure. This results in overall decrease of coarse-grained distribution function. In particular, values of coarse-grained distribution function cannot exceed the maximum value of the primordial distribution function [19]. The latter property enables one to obtain constraints on dark matter.

Primordial distribution function of sterile neutrinos produced via non-resonant oscilla-

tions can be approximated by scaled distribution of active neutrinos — ultra-relativistic Fermi-Dirac distribution [4],

$$f(p) = \frac{g}{(2\pi)^3} \frac{\beta}{e^{p/T_\nu} + 1} , \tag{1}$$

where $g = 2$ is the number of sterile neutrino spin states. The coefficient β here is proportional to the active-sterile mixing angle squared and can be related to the sterile neutrino mass by demanding that sterile neutrinos constitute all dark matter in the Universe[1]:

$$\beta = \left(\frac{\Omega_{\rm DM}}{0.2}\right)\left(\frac{10\,{\rm eV}}{m}\right) .$$

Thus, the maximum value of the primordial distribution function in this scenario is given by

$$\max f(p) = \frac{1}{(2\pi)^3}\left(\frac{\Omega_{\rm DM}}{0.2}\right)\left(\frac{10\,{\rm eV}}{m}\right) . \tag{2}$$

Observationally, the value of the coarse-grained distribution function in a galactic halo can be estimated by the phase-space density, the ratio between the mass density and cube of the one-dimensional velocity dispersion in a given volume, $Q \equiv \rho/\sigma^3$ [21]. In practice, one measures the velocity dispersion of stars and assumes that it coincides with that of dark matter particles. For non-relativistic dark matter particles one has

$$Q = m^4 \cdot \frac{n}{\langle \frac{1}{3} p^2 \rangle^{3/2}} ,$$

where n is their average number density in a halo. Assuming that the coarse-grained distribution of halo particles is isotropic, $f_{halo}(\mathbf{p}, \mathbf{r}) = f_{halo}(p, r)$, one estimates

$$\frac{n}{\langle p^2\rangle^{3/2}} = \frac{\left[\int f_{halo}(\mathbf{p},\mathbf{r}) d^3\mathbf{p}\right]^{5/2}}{\left[\int f_{halo}(\mathbf{p},\mathbf{r}) \mathbf{p}^2 d^3\mathbf{p}\right]^{3/2}} \sim f_{halo}(p_*, r) , \tag{3}$$

where p_* is a typical momentum of the dark matter particles.[2] In this way the value of the coarse-grained distribution function in a galactic halo is estimated as

$$f_{halo} \approx \frac{Q}{3^{3/2}\, m^4} . \tag{4}$$

Hence, one arrives at the following constraint on dark matter,

$$\frac{Q}{3^{3/2}\, m^4} < \max f(p) . \tag{5}$$

[1] We do not consider here the possibility that the non-resonant mechanism generates not all, but only a fraction of dark matter [20].

[2] In the case when the width of the momentum distribution around p_* is small, $\Delta p < p_*$, the estimate (3) reads $n/\langle p^2\rangle^{3/2} \sim f_{halo} \cdot (\Delta p/p_*)$. Then, instead of (4), one has an inequality, $f_{halo} > Q/(3^{3/2} m^4)$. This makes the constraint (5) even stronger.

The strongest constraints on warm dark matter scenario are obtained by making use of the highest values of the phase-space density in dark matter dominated objects. These are found in dwarf spheroidal satellite galaxies [22, 23]. dSph's are the most dark matter dominated compact objects observed so far, and are conjectured to be hosted by the smallest possible dark matter halos [22]. In recently discovered objects Coma Berenices, Leo IV and Canes Venaciti II, the value of Q ranges from $5 \cdot 10^{-3}\,\frac{M_\odot/\mathrm{pc}^3}{(\mathrm{km/s})^3}$ to $2 \cdot 10^{-2}\,\frac{M_\odot/\mathrm{pc}^3}{(\mathrm{km/s})^3}$ [24]. We use the first, slightly more conservative value,

$$Q \equiv q\,\frac{M_\odot/\mathrm{pc}^3}{(\mathrm{km/s})^3} = 5 \cdot 10^{-3}\,\frac{M_\odot/\mathrm{pc}^3}{(\mathrm{km/s})^3}\;. \tag{6}$$

Making use of Eqs. (2) and (5) we obtain the constraint on the sterile neutrino mass,

$$m > 5.7\ \mathrm{keV}\left(\frac{0.2}{\Omega_{\mathrm{DM}}}\right)^{1/3}\left(\frac{q}{5\cdot 10^{-3}}\right)^{1/3}\;. \tag{7}$$

Despite the fact that the value of observable Q may not be exactly the same as the value of the coarse-grained distribution function of dark matter particles, we consider the bound (7) as conservative. Indeed, the coarse grained distribution function is likely to decrease considerably during the non-linear stage of evolution. For example, some numerical simulations of halo formation show the decrease of Q by a factor of $10^2 - 10^3$ from input initial values [25]. Therefore, further improvement of understanding of how compact objects are formed by warm dark matter particles is likely to strengthen the bound (7). We conclude that the Tremaine–Gunn approach gives another argument, independent of Lyman-α , that disfavors the sterile neutrino dark matter generated by the non-resonant production mechanism.

However, sterile neutrino may still be a dark matter candidate. There are other generation mechanisms of sterile neutrino dark matter, such as resonant production in the presence of lepton asymmetry [5, 6], production in scattering [8] and production in decays [3, 7]. In these cases the sterile neutrino decay rate is not directly related to their abundance in the Universe, so the X-ray mass bound does not apply.

Although the primordial distribution function of sterile neutrinos in these scenarios differs from (1), lower bounds from both Lyman-α and phase-space observations are not expected to change dramatically. For example, the results presented in Ref. [6] indicate that the maximum value of the distribution function of 3 keV sterile neutrinos generated by the resonant production in the presence of lepton asymmetry is about $0.03-0.3$ of the maximum of thermal distribution, $1/(2\pi)^3$ (accounting for 2 spin states), depending on the value of the lepton asymmetry. In that case one has

$$3^{3/2}m^4\ \max f(p) = (6.5\cdot 10^{-3} - 6.5\cdot 10^{-2})\,\frac{M_\odot/\mathrm{pc}^3}{(\mathrm{km/s})^3}\;,$$

at the edge of the constraint (5), with Q estimated as in (6). In this case the correct dark matter density, $\Omega_\nu \approx 0.2$, is obtained for the mixing angle in the range $\sin^2(2\theta) = 10^{-11}-10^{-8}$ and even smaller, again depending on the lepton asymmetry, and the X-ray bound can be satisfied [6].

If sterile neutrinos are produced in scattering processes and do not equilibrate, as is the case for the production mechanism of Ref. [8], then the neutrino distribution is approximately given by (1) with the same coefficient β and effective temperature T_ν. Hence, the maximum of the neutrino distribution function is the same, as in the case of non-resonant production. So, the limit (7) is applicable for sterile neutrino produced in scattering as well.

Before proceeding to sterile neutrino production in decays, let us recall that there is a simple bound on their mass based on Pauli blocking. Namely, the primordial distribution function cannot exceed the value $2/(2\pi)^3$, so for distributions saturating this bound, the constraint (5) translates into

$$m > 1.0 \text{ keV} \left(\frac{q}{5\cdot 10^{-3}}\right)^{1/4} , \tag{8}$$

where we again used the estimate (6). This is a model-independent lower limit on the mass of a fermionic dark matter candidate (assuming two spin states). In what follows we do not take into account Pauli blocking, with understanding that if the limits obtained are weaker than (8), then the limit (8) applies.

Sterile neutrinos may be produced in decays of relativistic thermalized particles[3] of mass M and partial decay width at rest into sterile neutrinos Γ. In that case the low momentum part of sterile neutrino distribution function is given by [26, 3, 7, 27]

$$f(p) = \frac{8}{3}\frac{M_{\rm Pl}^*\Gamma}{M^2}\left(\frac{T_{0,eff}}{p}\right)^{1/2}\int\limits_0^\infty z^{3/2} f_{th}(z)\, dz = \frac{\zeta(5/2)}{4\pi^{5/2}}\;\frac{M_{\rm Pl}^*\Gamma}{M^2}\;\left(\frac{T_{0,eff}}{p}\right)^{1/2} . \tag{9}$$

Here $M_{\rm Pl}^* \equiv M_{\rm Pl}\sqrt{90/(8\pi^3 g_*)}$, $T_{0,eff} = \left(\frac{g_{*,0}}{g_*}\right)^{1/3} T_0$, where g_* and $g_{*,0}$ are the effective numbers of degrees of freedom at decay and present epoch, respectively, and f_{th} is the thermal distribution function of decaying particles. We assume here that the latter are scalars. Hereafter $f(p)$ denotes the primordial distribution function redshifted to the present epoch. The total present number density is [26]

$$n_0 = \frac{3\zeta(5)}{4\pi} T_{0,eff}^3 \;\frac{M_{\rm Pl}^*\Gamma}{M^2} .$$

[3]We assume here for simplicity that these particles interact sufficiently weakly with the rest of the cosmic plasma, so that sterile neutrino production in scattering processes is negligible. In the opposite case the mass bound is similar to (7).

Requiring that sterile neutrinos make all of dark matter, $n_0 m = \Omega_{\rm DM}\rho_c$, one finds the only relevant combination of parameters of decaying particles,

$$\frac{\Gamma}{M^2} = \frac{4\pi}{3\zeta(5)}\,\frac{\Omega_{\rm DM}\,\rho_c}{m\,M_{\rm Pl}^* T_{0,eff}^3}\ .$$

The distribution (9) is formally unbounded from above. In reality this means that at low momenta, the distribution function takes the Pauli blocking value, $f = 2/(2\pi)^3$. In this situation one in principle is still able to obtain mass limits stronger than (8) by invoking the following statistical argument [28]. One requires that in the early Universe, a certain fraction ν of dark matter particles are sufficiently densely packed in phase space so that these particles are able to form subsequently dark matter halos of high Q. In other words, the value of the distribution function of this fraction of particles should obey the constraint

$$f(p) > \frac{Q}{3^{3/2}\,m^4}\ , \tag{10}$$

where the observed value of Q is estimated as in (6). The fraction ν should not be smaller than the fraction of dark matter residing in dSph's, which we estimated in [26] as

$$\nu \sim 10^{-5}\ .$$

Given very weak dependence on ν in the limits we obtain in what follows, the precise number is unimportant for our purposes.

We continue the discussion of thermal creation by noticing that the fraction ν of most densely packed particles is related to the maximum momentum p_ν of these particles in an obvious way,

$$\int_0^{p_\nu} f(p)\,4\pi p^2 dp = \nu n_0\ .$$

Making use of (9) we obtain

$$\frac{p_f}{T_{0,eff}} = \left(\frac{15\sqrt{\pi}\,\zeta(5)}{8\,\zeta(5/2)}\ \nu\right)^{2/5}\ .$$

Substituting this value of momentum back into (9) we find

$$\begin{aligned} f(p_\nu) &= \frac{\Omega_{\rm DM}\,\rho_c}{m\,T_{0,eff}^3}\frac{1}{3\pi^{8/5}}\left(\frac{\zeta(5/2)}{\zeta(5)}\right)^{6/5}\left(\frac{8}{15}\right)^{1/5}\ \nu^{-1/5} \\ &= 1.1\cdot 10^{-2}\left(\frac{\Omega_{\rm DM}}{0.2}\right)\left(\frac{g_*}{106.75}\right)\left(\frac{1\,\mathrm{keV}}{m}\right)\left(\frac{10^{-5}}{\nu}\right)^{1/5}\ . \end{aligned}$$

It is this value that should obey the constraint (10). Hence, we obtain the lower bound on the sterile neutrino mass,

$$m > 0.88\,\mathrm{keV}\, \left(\frac{0.2}{\Omega_{\mathrm{DM}}}\right)^{1/3} \left(\frac{106.75}{g_*}\right)^{1/3} \left(\frac{q}{5\cdot 10^{-3}}\right)^{1/3} \left(\frac{\nu}{10^{-5}}\right)^{1/15} . \tag{11}$$

This bound is in fact slightly weaker than the Pauli blocking bound (8) for our values of parameters. As we noticed above, in this situation one should use the bound (8) instead.

The bound (11) is somewhat different from the bound obtained in Ref. [27], since we use the statistical approach rather than the approach of Ref. [21]. Notice that the dark matter is warm, i.e., the bound (8) is nearly saturated if the parameters M and Γ are such that

$$M\frac{M}{\Gamma} \sim 10^{19}\,\mathrm{GeV}\, \left(\frac{0.2}{\Omega_{\mathrm{DM}}}\right) \left(\frac{106.75}{g_*}\right)^{3/2} .$$

This is the case if either the decaying particles are very heavy or their decay rate into sterile neutrinos is very small, or both.

Finally, let us consider the production of sterile neutrinos in decays of non-relativistic particles whose number in comoving volume has been frozen out. In that case the momentum of sterile neutrinos at production equals $p_* = M/2$, and then the momentum gets redshifted to

$$p = p_* \frac{a(t)}{a(t_0)} = p_* \frac{T_{0,eff}}{T} , \tag{12}$$

where t is the time at decay and t_0 is the present time. Hence, the distribution function of sterile neutrinos is obtained from the relation

$$f(p)\, d^3p = n_0\, e^{-\Gamma_{tot}\cdot t}\, \Gamma_{tot}\, dt .$$

Notice that we normalized the distribution function to the present number density of sterile neutrinos, assuming that the abundance of decaying particles is just right to produce them. Therefore, this formula contains the total width Γ_{tot} only. Making use of (12) we obtain

$$f(p) = n_0\, e^{-\Gamma_{tot}\cdot t}\, \frac{\Gamma_{tot}}{H(t)}\, \frac{1}{4\pi p^3} . \tag{13}$$

Now, at radiation dominated epoch[4] $t = M_{\mathrm{Pl}}^*/(2T^2)$, so that from (12) we obtain

$$t = \frac{M^*_{\mathrm{Pl}}}{2T^2_{0,eff}} \left(\frac{p}{p_*}\right)^2 , \qquad H(t) = \frac{1}{2t} . \tag{14}$$

[4]One can check that the energy density of decaying non-relativistic particles never dominates in the case we consider here.

Hence, the distribution function (13) behaves as $1/p$ at low momenta, and we again have to employ the statistical argument. Time t_ν by which the fraction ν of sterile neutrinos is produced, is determined by

$$\nu = 1 - e^{-\Gamma_{tot} \cdot t_\nu} = \Gamma_{tot}\, t_\nu \ .$$

The corresponding momentum p_ν is found from (14). Inserting it into (13) and requiring that sterile neutrinos make all of dark matter, we find

$$\begin{aligned} f(p_\nu) &= \frac{\sqrt{2}}{\pi} \, \frac{\Omega_{\rm DM}\, \rho_c}{m\, T^3_{0,eff}} \left(\frac{M^*_{\rm Pl} \Gamma_{tot}}{M^2} \right)^{3/2} \nu^{-1/2} \\ &= 2.4 \, \left(\frac{M^*_{\rm Pl} \Gamma_{tot}}{M^2} \right)^{3/2} \left(\frac{\Omega_{\rm DM}}{0.2} \right) \left(\frac{g_*}{106.75} \right) \left(\frac{1\,{\rm keV}}{m} \right) \left(\frac{10^{-5}}{\nu} \right)^{1/2} \ . \end{aligned}$$

We again make use of (10) for this value of the distribution function, and obtain finally the bound

$$m > 145\,{\rm eV} \, \left(\frac{M^2}{M^*_{\rm Pl} \Gamma_{tot}} \right)^{1/2} \cdot \left(\frac{0.2}{\Omega_{\rm DM}} \right)^{1/3} \left(\frac{106.75}{g_*} \right)^{1/3} \left(\frac{q}{5 \cdot 10^{-3}} \right)^{1/3} \left(\frac{\nu}{10^{-5}} \right)^{1/6} \ . \qquad (15)$$

This bound should be used whenever it supersedes the bound (8).

Unlike the limit (11), the bound (15) depends on parameters M and Γ_{tot} characterizing the decaying particles. This is because unlike in the thermal production case, we now have (implicitly) one more free parameter, the number density of decaying particles at their freeze-out. Interestingly, for heavy enough decaying particles and/or long enough lifetime of these particles, the right hand side of (15) may be well above the "canonical" keV range. This implies that the decay mechanism we discuss here is capable of producing *warm* dark matter composed of *heavy* particles. As an example, for $M \simeq 10^{14}$ GeV, $\Gamma_{tot} = \frac{y^2}{8\pi} M$ and $y \simeq 10^{-12}$, sterile neutrino of mass in TeV range would be warm.

It is worth noting that the bounds (11) and (15) apply not only to sterile neutrinos but to any fermionic dark matter candidates produced in similar decay processes.

After this work has been completed, we received a draft of the paper [29], where similar issues have been considered. Our results are consistent with the results of Ref. [29] wherever they overlap.

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References

[1] T. Asaka and M. Shaposhnikov, Phys. Lett. B **620** (2005) 17 [arXiv:hep-ph/0505013]; M. Shaposhnikov, "Is there a new physics between electroweak and Planck scales?," [arXiv:0708.3550 [hep-th]].

[2] T. Asaka and M. Shaposhnikov, Phys. Lett. B **620** (2005) 17 [arXiv:hep-ph/0505013].

[3] M. Shaposhnikov and I. Tkachev, Phys. Lett. B **639** (2006) 414 [arXiv:hep-ph/0604236].

[4] S. Dodelson and L. M. Widrow, Phys. Rev. Lett. **72** (1994) 17 [arXiv:hep-ph/9303287]; A. D. Dolgov and S. H. Hansen, Astropart. Phys. **16** (2002) 339 [arXiv:hep-ph/0009083]; T. Asaka, M. Laine and M. Shaposhnikov, JHEP **0701** (2007) 091 [arXiv:hep-ph/0612182].

[5] X. D. Shi and G. M. Fuller, Phys. Rev. Lett. **82** (1999) 2832 [arXiv:astro-ph/9810076]; K. Abazajian, G. M. Fuller and M. Patel, Phys. Rev. D **64** (2001) 023501 [arXiv:astro-ph/0101524].

[6] M. Laine and M. Shaposhnikov, JCAP **0806** (2008) 031 [arXiv:0804.4543 [hep-ph]].

[7] A. Kusenko, Phys. Rev. Lett. **97** (2006) 241301 [arXiv:hep-ph/0609081]; K. Petraki and A. Kusenko, Phys. Rev. D **77** (2008) 065014 [arXiv:0711.4646 [hep-ph]]; K. Petraki, Phys. Rev. D **77** (2008) 105004 [arXiv:0801.3470 [hep-ph]].

[8] S. Khalil and O. Seto, "Sterile neutrino dark matter in $B-L$ extension of the standard model and galactic 511 keV line," arXiv:0804.0336 [hep-ph].

[9] A. Boyarsky, A. Neronov, O. Ruchayskiy, M. Shaposhnikov and I. Tkachev, Phys. Rev. Lett. **97** (2006) 261302 [arXiv:astro-ph/0603660]; S. Riemer-Sorensen, S. H. Hansen and K. Pedersen, Astrophys. J. **644** (2006) L33 [arXiv:astro-ph/0603661]; C. R. Watson, J. F. Beacom, H. Yuksel and T. P. Walker, Phys. Rev. D **74** (2006) 033009 [arXiv:astro-ph/0605424]; A. Boyarsky, J. Nevalainen and O. Ruchayskiy, Astron. Astrophys. **471** (2007) 51 [arXiv:astro-ph/0610961]; K. N. Abazajian, M. Markevitch, S. M. Koushiappas and R. C. Hickox, Phys. Rev. D **75** (2007) 063511 [arXiv:astro-ph/0611144]; A. Boyarsky, O. Ruchayskiy and M. Markevitch, Astrophys. J. **673** (2008) 752 [arXiv:astro-ph/0611168].

[10] A. Boyarsky, D. Iakubovskyi, O. Ruchayskiy and V. Savchenko, "Constraints on decaying Dark Matter from XMM-Newton observations of M31," [arXiv:0709.2301 [astro-ph]].

[11] P. Bode, J. P. Ostriker and N. Turok, Astrophys. J. **556** (2001) 93 [arXiv:astro-ph/0010389]; V. Avila-Reese, P. Colin, O. Valenzuela, E. D'Onghia and C. Firmani, Astrophys. J. **559** (2001) 516 [arXiv:astro-ph/0010525].

[12] G. Kauffmann, S. D. M. White and B. Guiderdoni, Mon. Not. Roy. Astron. Soc. **264**, 201 (1993); A. A. Klypin, A. V. Kravtsov, O. Valenzuela and F. Prada, Astrophys. J. **522**, 82 (1999) [arXiv:astro-ph/9901240]; B. Moore, S. Ghigna, F. Governato, G. Lake, T. Quinn, J. Stadel and P. Tozzi, Astrophys. J. **524**, L19 (1999); J. Diemand, M. Kuhlen and P. Madau, Astrophys. J. **657** (2007) 262 [arXiv:astro-ph/0611370].

[13] B. Moore, Nature **370** (1994) 629; W. J. G. de Blok, S. S. McGaugh, A. Bosma and V. C. Rubin, Astrophys. J. **552** (2001) L23 [arXiv:astro-ph/0103102]; J. D. Simon, A. D. Bolatto, A. Leroy, L. Blitz and E. L. Gates, Astrophys. J. **621** (2005) 757 [arXiv:astro-ph/0412035].

[14] J. Sommer-Larsen and A. Dolgov, Astrophys. J. **551** (2001) 608 [arXiv:astro-ph/9912166]; D. N. Chen and Y. P. Jing, Mon. Not. Roy. Astron. Soc. **336** (2002) 55 [arXiv:astro-ph/0201520]; M. Goetz and J. Sommer-Larsen, Astrophys. Space Sci. **284** (2003) 341 [arXiv:astro-ph/0210599].

[15] K. Abazajian, Phys. Rev. D **73** (2006) 063513 [arXiv:astro-ph/0512631]; U. Seljak, A. Makarov, P. McDonald and H. Trac, Phys. Rev. Lett. **97** (2006) 191303 [arXiv:astro-ph/0602430].

[16] M. Viel, G. D. Becker, J. S. Bolton, M. G. Haehnelt, M. Rauch and W. L. W. Sargent, Phys. Rev. Lett. **100** (2008) 041304 [arXiv:0709.0131 [astro-ph]].

[17] S. Tremaine and J. E. Gunn, Phys. Rev. Lett. **42** (1979) 407.

[18] C. Efthymiopoulos, N. Voglis and C. Kalapotharakos, "Special Features of Galactic Dynamics," [arXiv:astro-ph/0610246].

[19] D. Lynden-Bell, Mon. Not. Roy. Astron. Soc. **136** (1967) 101; S. Tremaine, M. Henon and D. Lynden-Bell, Mon. Not. Roy. Astron. Soc. **219** (1986) 285. P. H. Chavanis, "Statistical mechanics of violent relaxation in stellar systems," [arXiv:astro-ph/0212205].

[20] A. Palazzo, D. Cumberbatch, A. Slosar and J. Silk, Phys. Rev. D **76** (2007) 103511 [arXiv:0707.1495 [astro-ph]].

[21] C. J. Hogan and J. J. Dalcanton, Phys. Rev. D **62** (2000) 063511 [arXiv:astro-ph/0002330].

[22] M. Mateo, Ann. Rev. Astron. Astrophys. **36** (1998) 435 [arXiv:astro-ph/9810070].

[23] J. J. Dalcanton and C. J. Hogan, Astrophys. J. **561** (2001) 35 [arXiv:astro-ph/0004381].

[24] J. D. Simon and M. Geha, Astrophys. J. **670** (2007) 313-331 [arXiv:0706.0516 [astro-ph]].

[25] S. Peirani, F. Durier and J. A. De Freitas Pacheco, Mon. Not. Roy. Astron. Soc. **367** (2006) 1011 [arXiv:astro-ph/0512482]; S. Peirani and J. A. de Freitas Pacheco, "Phase-Space Evolution of Dark Matter Halos," [arXiv:astro-ph/0701292].

[26] D. Gorbunov, A. Khmelnitsky and V. Rubakov, "Is gravitino still a warm dark matter candidate?," [arXiv:0805.2836 [hep-ph]].

[27] D. Boyanovsky, "Clustering properties of a sterile neutrino dark matter candidate," [arXiv:0807.0646 [astro-ph]].

[28] J. Madsen, Phys. Rev. D **44** (1991) 999; J. Madsen, Phys. Rev. D **64** (2001) 027301 [arXiv:astro-ph/0006074].

[29] A. Boyarsky, O. Ruchayskiy and D. Iakubovskyi, "A lower bound on the mass of Dark Matter particles," [arXiv:0808.3902 [hep-ph]].